Symposium of the International Association for Boundary Element Methods 2024

Proceeding

Hong Kong, Dec. 4-6 2024

https://iabem2024hkust-dev.hkust.edu.hk

Organizers: Wenjing Ye, Yijun Liu

Preface

Dear colleagues

It is a great pleasure to have an IABEM symposium again after 6 years of break since the Paris symposium in 2018. As a matter of fact, the symposium in Hong Kong was originally scheduled in 2020. Unfortunately, however, we had to postpone it due to COVID-19 and a situation in Hong Kong. Indeed, COVID-19 was a big blow to the academic world, and we have seen efforts to have meetings on-line during the pandemic. Some of such attempts seem to have worked out reasonably well, but we have chosen to stick to the conventional in-person format, which we believe to be suitable for a small, but open and free organization like IABEM where direct communications are essential. Therefore, this Hong Kong symposium is really a long-waited one.

This volume includes abstracts of papers presented in this symposium, which show new developments of our time-honoured but ever growing computational methodology, i.e., BEM. We hope that these papers are inspiring to the reader and lead to new ideas.

These 6 years have seen many changes. The world does not seem to be as peaceful as it was when I agreed to serve as the president in Paris in 2018. I was sure that I could complete the 2 year term without problem, then. Due to the pandemic, however, my presidency became much longer than expected, during which period I retired from academia and became inactive in BEM. I feel sorry for not having been able to do anything other than postponing the symposium as the president of IABEM. Fortunately, Prof. Ralf Hiptmair, an executive council member, volunteered to serve as an acting president. I would like to thank Prof. Hiptmair for his leadership and other EC members for their support. Finally, but not least, I would like to thank the chair and co-chair of this symposium, Prof. Wenjing Ye and Prof. Yijun Liu, without whose painstaking preparation this symposium was impossible.

Naoshi Nishimura (Professor Emeritus, Kyoto University and President of IABEM)

Acknowledgement

We extend our gratitude to the following organizations for their support of this symposium, with special acknowledgment of the financial assistance provided by the Department of Mechanical and Aerospace Engineering at the Hong Kong University of Science and Technology.

IABEM

香港科技大學
THE HONG KONG UNIVERSITY OF SCIENCE AND TECHNOLOGY

SUSTech
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Conference Venue: Mr and Mrs Lee Siu Lun Lecture Theater K (LT-K)

Registration Venue: outside LT-K

Lecture Theaters and Lift Location
The Hong Kong University of Science and Technology

Wi-Fi and Internet Access

There is a public Wi-Fi access available under the name “Wi-Fi.HK via HKUST”, which is open to all users. This network covers the majority of the campus areas of Hong Kong University of Science and Technology.

Upon registration, you will receive a dedicated Wi-Fi account for more reliable access. Detailed information about this account will be provided at that time.

On-campus Restaurants

Please refer to https://cso.ust.hk/locations/restaurants for a list of on-campus restaurants, canteens and coffee shops along with their respective locations and opening hours.

Program at a Glance

<table>
<tr><td>Tuesday
Dec. 3</td><td></td><td>Wednesday
Dec. 4</td><td>Thursday
Dec. 5</td><td>Friday
Dec. 6</td></tr>
<tr><td rowspan="2"></td><td rowspan="2">8:00am –
5:00pm</td><td>Registration</td><td rowspan="2">Registration</td><td rowspan="2">Registration</td></tr>
<tr><td>Opening Speech and Award Ceremony (8:45am – 9:00am)</td></tr>
<tr><td rowspan="9">Registration
6:00pm –
8:30pm</td><td>9:00am –
9:50am</td><td>Rizzo Lecture</td><td rowspan="2">Technical
Session</td><td rowspan="2">Technical
Session</td></tr>
<tr><td>9:50am -
10:15am</td><td>Technical Session</td></tr>
<tr><td>10:15am –
10:45am</td><td>Coffee break</td><td>Coffee break</td><td>Coffee break</td></tr>
<tr><td>10:45am –
12: 00pm</td><td>Technical Session</td><td>Technical
Session</td><td>Technical
Session</td></tr>
<tr><td>12:00pm –
2:00pm</td><td>Lunch</td><td>Lunch</td><td>Lunch</td></tr>
<tr><td>2:00pm –
3:15pm</td><td>Technical Session</td><td>Technical
Session</td><td>Technical
Session</td></tr>
<tr><td>3:15pm –
3:45pm</td><td>Coffee Break</td><td>Coffee Break</td><td rowspan="4">Social
Activity</td></tr>
<tr><td>3:45pm –
5:25pm</td><td>Technical Session</td><td>Technical
Session</td></tr>
<tr><td>5:25pm –
6:00pm</td><td>Assembly meeting</td><td></td></tr>
<tr><td></td><td>6:00pm</td><td></td><td>Banquet</td></tr>
</table>

Wednesday December 4 2024

Morning session

08:45 – 08:50 Welcome speech (R. Hiptmair)

08:50 – 09:00 Rizzo Award Ceremony (Chairperson Y. Liu)

Chairperson R. Hiptmair

09:00 – 09:50 **Rizzo Award Lecture** Simon Chandler-Wilde: *First kind BIEs and BEM for acoustic scattering*

09:50 – 10:15 P.G. Martinsson: *Fast direct solvers for boundary integral equations*

10:15 – 10:45 Coffee Break

Chairperson: R. Hiptmair

10:45 – 11:10 M. Schanz: *FMM and H-matrix based approaches within a 3D-ACA accelerated time domain boundary element method*

11:10 – 11:35 Y. Matsumoto, T. Maruyama: *A fast direct solver for time-harmonic in-plane elastic wave scattering problems*

11:35 – 12:00 Z.Y. Gao, Z.L. Li, Y.J. Liu: *An efficient time-domain BEM using a kernel-function library for 3D acoustic problems*

Afternoon session

Chairperson: Yijun Liu

14:00 – 14:25 T. Kramer, B. Marussig, M. Schanz: *A higher order time domain boundary element formulation based on isogeometric analysis and the convolution quadrature method*

14:25 – 14:50 C. Schwarz: *Collocation and a mixed approximation of the boundary element method for linear elasticity*

14:50 – 15:15 V.L. Keshava, M. Schanz: *Partial integration based regularization in fast multipole boundary element method*

15:15 – 15:45 Coffee Break

Chairperson: Yijun Liu

15:45 – 16:10 D. Seibel: *Calculation of singular integrals in Galerkin boundary element methods*

16:10 – 16:35 H.B. Chen, J.L. Zhang, X.F. Xiong: *FE-BE based multi-scale topology optimization approach for exterior vibro-acoustic interaction system*

16:35 – 17:00 X.H. Lin, H.B. Chen: *Robust topology optimization in fully coupled vibro-acoustic systems with random and interval variables based on FEM-BEM*

17:00 – 17:25 R. Toshimitsu, H. Isakari: *Structural optimization for acoustic cavity problems in half space using a hybrid boundary integral representation*

Thursday December 5 2024

Morning session

Chairperson: M. Schanz

09:00 – 09:25 R. Hiptmair, P. Panchal: *BEM for electromagnetic forces: a shape calculus approach*

09:25 – 09:50 J.C. Zhou, E. Pan, Z.Q. Zhang: *Synthetic seismograms in liquid-solid coupled layered media*

09:50 – 10:15 M. Faustmann, A. Rieder: *FEM-BEM coupling in fractional Diffusion*

10:15 – 10:45 Coffee Break

Chairperson: M. Schanz

10:45 – 11:10 S. Chaillat, M. Darbas, P. Escapil-Inchauspé, C. Jerez-Hanckes: *Local Multiple Traces Formulation for Elastic Wave Scattering by Heterogeneous Scatterers*

11:10 – 11:35 D. Hoonhout, R. Löscher, O. Steinbach, C. A. Urzúa-Torres: *Stable adaptive least-squares space-time BEM for the wave equation*

11:35 – 12:00 K. Matsushima, T. Yamada: *Identification of non-Hermitian degeneracy in acoustic scattering systems using boundary integral equations*

Afternoon session

Chairperson: S. Chaillat

14:00 – 14:25 K. Tomoyasu, H. Isakari: *Performance evaluation of the Calderon preconditioning for the Burton–Miller boundary element method in three-dimensional transmission problems*

14:25 – 14:50 P. Gélat, R. Haqshenas, E. van 't Wout: *Flexible boundary integral formulations for multiple and nested domains*

14:50 – 15:15 T. Maruyama, H. Shoji: *Fundamental study on numerical homogenization for wave problems using self-consistent and boundary element methods*

15:15 – 15:45 Coffee Break

Chairperson: H. Chen

15:45 – 16:10 D. Sun, X.W. Gao, T.Y. Qin: *Analysis for Complex Plane Cracks Using the Singular Integral Equation Method*

16:10 – 16:35 Y. Yang, X. Wei and Y. Liu: *A multi-scaled fracture analysis method based on BEM and PD*

16:35 – 17:00 W.Z. Xu, Z.J. Fu: *Fast acoustic singular boundary method based on directional* H^2*-matrices*

17:00 – 17:25 Z.J. Fu, Q. Xi, W.Z. Xu: *Some recent development of Singular boundary method*

Friday December 6 2024

Morning session

Chairperson: H. Isakari

09:00 – 09:25 S. Chaillat, I. McBrearty, E. Dunham: *Stable simulations of non-spherical bubble of gas with Boundary Element Methods*

09:25 – 09:50 Z. Farooq, A. Pashov, G. Degrande, S. François: *Proper Generalized Decomposition applied to an elastodynamic boundary element formulation*

09:50 – 10:15 R.Y. Li, W. Ye, Y.J. Liu: *Decomposed B-FNO for 3D parametric acoustic wave analysis*

10:15 – 10:45 Coffee Break

Chairperson: X.W. Gao

10:45 – 11:10 N.A. Dumont: *Consistent, geometry-preserving boundary element method for 2D problems considering homothetic meshes*

11:10 – 11:35 V. Chien Le, K. Cools: *A boundary element method for the magnetic field integral equation in electromagnetics*

11:35 – 12:00 J.W. Lee, Y.S. Hiesh, S.K. Kao, J.T. Chen: *Application of boundary integral quadrature method to torsion problems of the orthotropic bars and its treatment of degenerate scale problem*

Afternoon session

Chairperson: W. Ye

14:00 – 14:25 I. Marchevsky, G. Shcheglov, E. Ryatina: *T-scheme for solution of the boundary integral equations in vortex methods for three-dimensional flows simulation*

14:25 – 14:50 A. Kolganova, I. Marchevsky: *Fast algorithm for solving boundary integral equations in vortex methods*

14:50 – 15:15 Y. Izmailova, I. Marchevsky: *Numerical schemes for the solution of the boundary integral equation arising in vortex methods for two-dimensional flow simulation around airfoils*

Abstracts
(Alphabetical order)

Fast algorithm for solving boundary integral equations in vortex methods

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Keywords: vortex methods, boundary integral equation, fast algorithms

Currently, vortex particle methods [1], belonging to the class of meshless Lagrangian methods of CFD, are an efficient tool for solving problems arising in engineering applications, mainly connected with estimating of hydrodynamic loads acting on airfoils in liquid or gas flow. They also make it possible to perform numerical simulation in fluid-structure interaction (FSI) problems when an airfoil is movable or deformable under aerohydrodynamic loads.

The idea of vortex methods is to consider vorticity field as primary computational variable. The velocity and pressure fields can be reconstructed according to the generalized Helmholtz decomposition and the generalized Cauchy — Lagrange integral. There is also the possibility of computation of integral force and momenta acting on the airfoil, as well as viscous friction forces.

The algorithm of vortex method includes several operations at each time step: solving of the boundary integral equation that describes vorticity generation at the airfoil surface; solving of N - body type problems, as well as performing number of other actions related to the calculation of particle interaction. “Direct” algorithms for these problems have quadratic computational complexity with respect to number of particles, that significantly bounds the applicability of vortex methods for solving actual problems in which number of vortex particles can reach order of a million for detailed flow representation. Solving the boundary integral equation becomes especially time-consuming for modeling a system of airfoils which are movable relative to each other.

In the proposed presentation fast algorithms of quasi-linear computational complexity are developed and implemented, which are based on well-known approaches (Barnes — Hut and Fast Multipole methods), and on some original author’s modifications [2]. The suggested algorithms are optimized especially for 2D vortex methods; the most accurate numerical approaches are implemented: the Galerkin-type schemes with piecewise-constant and piecewise-linear solution representation, including the possibility of singularity reconstruction in corner point.

Implementations of the developed fast algorithms are presented for both central processors (CPUs) and graphics accelerators (GPUs).

Thanks to the implementation of fast methods, it is possible now to perform unsteady simulations within hundreds of seconds of “physical” time, that requires only a few hours of machine time (at least for CUDA version for GPU). The results obtained for model problems of flutter critical wind speed determining are in good agreement with the experimental and calculated data of other authors.

The calculations were performed in the authors original open-source VM2D code [3], which allows one to simulate the flow of a viscous incompressible medium and is based on the modern modifications of vortex methods.

References

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FE-BE based multi-scale topology optimization approach for exterior vibro-acoustic interaction system

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Keywords: Multi-scale design, Microstructural topology optimization, Exterior vibro-acoustic inter- action systems, Piecewise constant level set method

This research presents a concurrent multi-scale topology optimization approach of bi-material structure for minimizing the response of exterior vibro-acoustic interaction system. The concurrent optimization simultaneously designs the microstructures and their distribution in the macroscopic structural domain to mitigate the response of vibro-acoustic system with infinite acoustic domain. The coupling algorithm is adopted by combining the finite element (FE) structural modeling and boundary element (BE) acoustic field modeling to analyze the response of vibro-acoustic system. The macrostructure in designed area is assumed to be composed of periodic microstructures and the equiv- alent material properties of the macrostructure are calculated by the homogenization method. The topology optimization model is schemed based on the piecewise constant level set method and the coupling interface between the structure and the acoustic domain is assumed to be unchanged during the design process. A new updating strategy of the penalty coefficient based on the sensitivity infor- mation of the objective function is adopted to overcome its problem-dependence. Numerical results show that the responses of the investigated coupled systems reduce significantly after optimization process, indicating the effectiveness of the proposed optimization algorithm, and the simultaneous optimization design at two scales yields better results compared to single-scale microstructural design.

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[3] Z. Zhang, W. Chen, An approach for topology optimization of damping layer under harmonic excitations based on piecewise constant level set method, *Journal of Computational Physics* **390** (2019), pp. 470-489.

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First kind BIEs and BEM for acoustic scattering

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Keywords: Boundary integral equations, Acoustic scattering

We consider the so-called sound-soft problem in time-harmonic acoustic scattering, where the total field, a solution to the Helmholtz equation in the exterior of some obstacle, vanishes on the boundary of the obstacle. Making an ansatz for the scattered field as a single-layer potential with some unknown density, a BIE for the density is obtained by applying this sound-soft boundary condition. This is an old formulation; it can be found, in the context of scattering by screens and apertures, already in 19th century texts [1]. In this talk we make a survey of previous results, we explain in what sense this formulation applies when the obstacle is some arbitrary compact set, and, in this general context, we establish wavenumber-explicit bounds on condition numbers [3,4]. In the case when the scattering obstacle is a self-similar fractal of some fractal dimension d, we implement and establish convergence rates for a fully discrete Galerkin BEM where all integrals are with respect to d-dimensional Hausdorff measure [2,3].

References:

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Stable simulations of non-spherical bubble of gas with Boundary Element Methods

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Keywords: Laplace problems, fast BEM, Neural Networks

Gas-filled bubbles are present in various fields of science and engineering, such as medicine, life sciences, ocean engineering, mechanical engineering, and materials engineering. The phenomenon of bubble dynamics, particularly the succession of expansion and collapse phases due to pressure imbalances, is crucial for numerous applications. For example, acoustic cavitation bubbles, typically millimeter or micrometer-sized, are used in sonoluminescence, ultrasound therapy, and drug delivery. This work focuses on underwater explosion bubbles and air-gun bubbles, which are meter-sized and used in geophysical explorations.

The simulation of bubble dynamics presents various numerical challenges due to the need to consider different scales, various boundaries, and multiple oscillation cycles. Various models have been proposed in the literature. The simplest one is the Rayleigh-Plesset equation, which describes the collapse of a bubble in an incompressible flow using a single ordinary differential equation (ODE). Recently, more complex models have been proposed by Zhang et al. [1]. The main assumption in all these models is that the bubble remains spherical throughout the simulation. To consider non-spherical bubbles in the context of incompressible flow, the boundary element method (BEM) can be used. At each time step, an exterior Laplace problem with Dirichlet boundary conditions is solved to update the bubble's geometry, with the boundary condition being updated using the Bernoulli equation.

However, this problem is much more complicated than it appears. There are two primary difficulties. The first difficulty is that the resolution is very costly because, at each time step, a Laplace problem must be solved on an evolving geometry. Even though fast BEMs are very efficient and can be used [2], they become too costly in this problem as soon as the resolution time exceeds one second, which limits accuracy. The second difficulty is that the problem is subject to numerical instabilities due to approximations at each BEM resolution. In this presentation, I will explain why this problem is unstable and how we plan to use neural networks to speed up the solution. These networks will be optimally designed using our knowledge of fast BEM [3] and tailored to evolving geometry over time.

References

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Local Multiple Traces Formulation for Elastic Wave Scattering by Heterogeneous Scatterers

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Keywords: multiple traces formulation, elastic wave scattering, operator preconditioning,

Context

The so-called (local) Multiple Trace Formulation (MTF) was originally introduced in [1] to tackle acoustic or Helmholtz wave propagation transmission problems over multiple heterogeneous domains Ω_m in two- and three-dimensions [3] with extensions to the electromagnetic case in [3]. The key elements behind MTFs are the following:

(i) For each subdomain Ω_m, the interior Dirichlet and Neumann boundary pair $(\lambda_D^m, \lambda_N^m)$ is treated as independent unknowns. These unknowns satisfy the corresponding integral representation formula for the solution inside the domain, i.e., if there are no volume forces and $\mathcal{S}_m$ and $\mathcal{D}_m$ denote the single and double layer potentials, respectively, then we can write

$$u = \mathcal{D}_m \lambda_D^m - \mathcal{S}_m \lambda_N^m, \text{in } \Omega_m.$$

(ii) Transmission conditions across each interface are considered only in terms of the interior Cauchy pair and enforced weakly by testing with suitable functions. Consequently, instead of using the exterior minus interior traces of a given domain Ω_m at its interface with Ω_n, we reorient the interior Neumann trace of the adjacent domain, i.e. $\gamma_N^{m,c} u = -\gamma_N^n u$ at the $\Omega_m \cap \Omega_n$.

Contribution

In this work, we will extend the above method to the case of elastic wave scattering in two and three dimensions. In particular, we will address how the method behaves when the corresponding Neumann or traction conditions are subject to changing Lamé parameter jumps. We also explain how does one constructs the elastic hypersingular operator following [4] as well as how to precondition the resulting MTF operator system as in [5].

References

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Consistent, geometry-preserving boundary element method for 2D problems considering homothetic meshes

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Keywords: Consistent BEM, geometry-preserving approach, homothetic mesh refinement

The collocation boundary element method, as developed and taught in the traditional books, suffers from severe inconsistencies. This has been recently corrected, as summarized in the review paper [1], in which we report the proposition of a convergence theorem for the general, just consistent, three- dimensional isoparametric formulation of potential and elasticity problems [2]. We have implemented, for both two-dimensional potential and elasticity problems, real-variable and – still better – complex-variable [3] codes for generally high-order, curved elements, which effortlessly enable evaluations with numerical precision that is only machine-limited and only resorts to the problem's mathematics – plus Gauss-Legendre quadrature of all integrals' regular parts. Despite the singular kernels that enter diverse forms of Somigliana's identity (as for evaluating stress results at a domain point), problem- unrelated singularities do not come up in a mathematically consistent formulation and implementation: singularities are just man-made. A few examples of highly challenging topological issues are presented. We show that fracture mechanics problems may be dealt with seamlessly, and we manage to objectively assess a problem's topological consistency, precision, accuracy, and round-off errors related to a given mesh discretization and software-conditioned computational digits. On the other hand, an isoparametric formulation may fail to reproduce the exact geometry of the idealized physical problem.

In the present contribution, further to addressing, as before, the three entities – *boundary nodes n* (for potentials or displacements), *boundary loci* ℓ (to which normal fluxes or tractions are referred), and *domain points s*, at which we collocate the singular sources – we propose a *geometry-preserving* concept for the simulation of real-world problems. The convergence theorem of [2] is no longer applicable. However, this is compensated by the improved geometric description of a physical model. Moreover, nodes and elements are placed and the mesh adaptively refined along a boundary patch in the frame of a *homothetic* concept. This avoids distortions in the problem's geometry description and leads to more robust simulations and more accurate and reliable results. Also, instead of working element by element, we only need to evaluate the complex location of a patch-related natural coordinate ζ_s corresponding to a given source point (x_s, y_s). This and the simplifications related to the homothetic mesh refinement, particularly using the complex $z = x + iy$, lead to significantly smaller computation time of all relevant quantities than in the original codes reported in [1]. The analytical, correction terms to be accrued for quasi-singularities are the same ones previously proposed.

References

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FEM-BEM coupling in Fractional Diffusion

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Keywords: Fractional operators, full-space problem, FEM-BEM coupling

In this talk, we consider fractional differential equations posed on the full space $\mathbb{R}^d$. A distinct advantage of full-space formulations for fractional PDEs is that all common definitions of non-integer powers of differential operators are equivalent, which is not true for formulations on bounded domains.

In order to treat the full-space problem numerically, several reformulations have to be made. Starting with the well-known Caffarelli-Silvestre extension to $\mathbb{R}^d \times \mathbb{R}^+$, we truncate the extension problem in the extended variable only to $\mathbb{R}^d \times (0, \mathcal{Y})$ for some $\mathcal{Y} > 0$. Then, a diagonalization procedure similar to [1] can be employed that leads to a sequence of scalar Helmholtz-type problems, which are discretized with a symmetric coupling of finite elements and boundary elements. Combined with a hp-FEM discretization in the extended variable, this gives a fully computable approximation with reasonable computational effort.

For the mentioned reformulations, inspired by [4], we show well-posedness in certain exotic Hilbert spaces. Using purely variational techniques, we derive an algebraic rate of decay of the solution of the truncated problem to the full-space solution as $\mathcal{Y}\to\infty$ as well as estimates of weighted analytic type for higher order derivatives of the truncated extension problem. These decay and regularity estimates can be used to derive a-priori estimates for the error between the exact full-space solution and an approximation based on our approach using a coupling of finite elements and boundary elements.

References

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Proper Generalized Decomposition applied to an elastodynamic boundary element formulation

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Keywords: Boundary element method, Green's functions, Proper Generalized Decomposition, model order reduction

Solving elastodynamic problems in an unbounded soil domain is often necessary in various civil engineering applications, such as the prediction of environmental vibration and seismic waves, and the boundary element (BE) method proves to be a robust approach for these dynamic problems. The fundamental solutions or Green's functions are an indispensable ingredient in the BE formulations. They reduce the complexity of the numerical solution of boundary integral equations along with the need for extensive boundary discretization and integration over complex geometries. However, for large-dimensional problems involving a layered halfspace and numerous source-receiver configurations across a wide frequency spectrum, calculating Green's functions remains computationally demanding. The fast-multipole method (FMM) [1] and methods using hierarchical ($\mathscr{H}$-) matrices [2] have been developed to optimize the solution process. Despite these advancements, computational bottlenecks persist in the fast multipole decomposition of Green's functions for the FMM-BE formulation and the computation of numerical Green's functions used in the $\mathscr{H}$-BE algorithm.

Proper Generalized Decomposition (PGD) [3, 4] is adopted here to derive Green's functions in a separated/decomposed form as an input for the BE method. The PGD formulation is an a priori model order reduction technique and is based on the assumption of a separable form of the multi-dimensional field. Each separated component therefore consists of a rank-one tensor that is computed iteratively in a greedy manner. Source and receiver positions, frequency, wavenumber, soil and foundation parameters are included as coordinates in the PGD formulation. By separating source and receiver terms, this study demonstrates how the PGD provides an alternative to multipole Green's functions.

The low rank approximations for Green's functions are introduced in the boundary integral equation, transforming the high-fidelity complex system into a series of decoupled one-dimensional vectors. This modified boundary element formulation is solved using iterative solvers, separating components such as source, frequency, and soil/foundation properties without needing boundary integration. This reduced order formulation of the integral equation transforms the original full order matrix-vector products, into vector-vector operations, resulting in substantial improvements in efficiency. The application of the PGD-BE formulation is demonstrated through examples of surface and embedded foundations on a layered halfspace. The solution is compared to the classical BEM formulation, implemented using the BEMFUN toolbox [5], and other fast BE approaches mentioned earlier. Conclusions emphasize the comparative analysis of solution accuracy and computational efficiency in term of memory requirements and CPU time between PGD-BE and other BE approaches.

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Some recent development of Singular boundary method

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Keywords: Singular boundary method, Non-uniqueness, Moment condition, Combined Helmholtz integral equation formulation

This study presents some recent development of the singular boundary method. The basic concepts of the singular boundary method are first introduced. Then the approaches to determine the origin intensity factors are classified and presented. Non-uniqueness issue encountered in the SBM are solved by moment condition and modified combined Helmholtz integral equation formulation (CHIEF). Several selected numerical examples are presented and discussed to show the recent developments and applications of the SBM to the solution of some typical boundary value problems, such as anomalous heat conduction, high wavenumber acoustic wave propagation, water wave-structure interaction, elastic wave propagation and structural vibration induced underwater acoustic radiation in the shallow ocean environment and so on.

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Flexible Boundary Integral Formulations for Multiple and Nested Domains

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Keywords: boundary integral formulations, acoustics, preconditioning

The boundary element method (BEM) efficiently solves acoustic wave propagation in unbounded domains since it automatically satisfies the radiation conditions at infinity. Another competitive ad- vantage of using the free-space Green's functions in the BEM is the accuracy of simulations achieved at high frequencies with only moderate mesh element sizes. However, the BEM's dependence on Green's functions limits its scope of feasible material heterogeneity. Here, we consider acoustic propagation through a general class of domains. Specifically, it may include combinations of disjointed and nested regions as long as each has constant material parameters.

There are many options to design a boundary integral formulation for acoustic wave propagation through penetrable domains [1]. The choice of how to rewrite the volumetric Helmholtz equation into a boundary integral formulation impacts the BEM's efficiency, accuracy and convergence. Furthermore, the efficacy of preconditioning strategies strongly depends on the formulation type [2]. In this study, we present a general framework to design boundary integral formulations of the Helmholtz equation at multiple and nested geometries, which allows for flexibility in combining different formulations at different material interfaces.

We implemented various preconditioned boundary integral formulations in our open-source Python library, OptimUS (https://github.com/optimuslib), utilising the BEMPP library as the computational backend. The methodology is validated against analytical solutions on concentric spheres and tested on various transmission problems. These extensions of the BEM's capabilities to accommodate more diverse geometrical settings are essential for various biomedical engineering applications [3, 4]. Specifically, we simulated acoustic propagation through complex domains with characteristic sizes of more than one hundred wavelengths. Among these large-scale simulations, we mention focused ultrasound targeting a kidney enclosed by perinephric fat, neuromodulation with transcranial ultrasound, and environmental noise propagation inside an abdominal model during pregnancy. Our innovative frame- work can easily be extended with more preconditioners and formulations to enhance the BEM's ability to simulate large-scale applications that involve acoustic wave propagation.

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An efficient time-domain BEM using a kernel-function library for 3D acoustic problems

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Keywords: Acoustic waves, Time-domain BEM, Kernel-function library, Transient response

Time-domain boundary element method (TDBEM) can be applied in studying transient acoustic wave problems. However, in order to avoid calculations of coefficients at different time steps, TDBEM usually requires storing or recomputing the coefficients at each time step. This can lead to significant amount of memory storage or long computing time. In this talk, an acoustic TDBEM based on a kernel-function library (KFL-BEM) is presented, in order to reduce the memory consumption of the time-domain conventional BEM (CBEM) and speedup the computation. In this approach, the storage requirement is reduced from $O(N^2 N_{tmin})$ to $O(N^2)$ where N represents the number of degrees of freedom of the model, and N_{tmin} is the minimum number of time steps for which coefficients need to be computed and stored. To demonstrate the effectiveness of the KFL-BEM, two verification examples are presented using the problems of a pulsating sphere and sound propagating in a channel. Compared with the CBEM, the KFL-BEM can significantly reduce the memory storage size as it does not require storing coefficients for any previous time steps. This method can also be applied to solve vibro-acoustic problems in the time domain. As an example, the acoustic radiation responses of a tuning fork under different striking loads are studied using the FEM and the proposed KFL-BEM, which clearly shows the potentials of the KFL-BEM in solving time-domain acoustic problems.

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Stable adaptive least-squares space-time BEM for the wave equation

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Keywords: Wave equation, Boundary Element Methods, Space-time, Least-squares

We consider space-time boundary element methods for the weakly singular operator V corresponding to transient wave problems. In particular, we restrict ourselves to the one-dimensional case and work with prescribed Dirichlet data and zero initial conditions. We begin by revisiting two approaches: energetic BEM [1] and the more recent formulation proposed in [3], for which the weakly singular operator is continuous and satisfies inf-sup conditions in the related spaces. However, numerical evidence suggests that it is unstable when using low-order Galerkin-Bubnov discretisations. As an alternative, it was shown in [4] that one obtains ellipticity -and thus stability- by composing V with the modified Hilbert transform [5].

In this talk, we reformulate these variational formulations as minimisation problems in L2. For discretisation, the minimisation problem is restated as a mixed saddle point formulation. Unique solvability can be established by combining conforming nested boundary element spaces for the mixed formulation such that the first-kind variational formulation is discrete inf-sup stable. We will analyse under which conditions the discrete inf-sup stability is satisfied, and, moreover, we will show that the mixed formulation provides a simple error estimator, which can be used for adaptivity. The theory is complemented by several numerical examples.

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BEM for Electromagnetic Forces: A Shape Calculus Approach

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We are concerned with the numerical computation of electrostatic forces/torques in only piece- wise homogeneous materials using the boundary element method (BEM). Conventional force formulas based on the Maxwell stress tensor yield functionals that fail to be continuous on natural trace spaces. Thus their use in conjunction with BEM incurs slow convergence and low accuracy.

Our starting point is the virtual work principle and its rigorous mathematical treatment based on the tools of shape calculus. We regard the force field as as the shape gradient of the field energy. We express the latter through traces of fields on interfaces and boundaries. Those traces are also the unknowns in the boundary integral equations (BIEs) underlying the boundary element method (BEM). Thus we have to differentiate a BIE-constrained functional on trace spaces.

This can be tackled using the adjoint method of constrained optimization in combination with a transformation of the BIEs to a reference geometry. Thus we can derive interface/boundary- based force formula that solely rely on field traces and are continuous on energy trace spaces. Thus, when evaluated for Galerkin BEM approximations of field traces, they enjoy superior accuracy and faster convergence compared to conventional formulas.

The same considerations can also be applied to the computation of magnetic forces. This is work in progress.

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Numerical schemes for the solution of the boundary integral equation arising in vortex methods for two-dimensional flow simulation around airfoils

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Keywords: vortex methods, boundary integral equation, Galerkin approach, *T*-schemes

Lagrangian meshless vortex methods [1] are a powerful tool in CFD for simulating incompressible flows, in particular, for estimating hydrodynamic loads acting on airfoil or system of airfoils in the flow. Lagrangian particles that move in inviscid or viscous medium are vorticity carriers. One of modern modifications of vortex methods, namely the Viscous Vortex Domain (VVD) method [2], developed by G.Ya. Dynnikova, is considered. It is implemented by authors in the VM2D code [3] that allows for sufficiently accurate flow simulation around airfoils at low Reynolds numbers.

In vortex methods, vorticity is the primary computational variable. However, the no-slip boundary condition (BC) cannot be explicitly formulated in terms of vorticity, so and it remains in traditional form for velocity being, in turn, reconstructed through integral over vorticity. Thus, the BC is satisfied due to vorticity flux from the airfoil boundary into the flow domain. Its intensity is represented implicitly through the vortex sheet intensity which satisfies to the boundary integral equation (BIE).

Traditionally, in common implementations of vortex methods singular or hypersingular BIEs are considered. Numerical schemes for such equations are typically some types of collocation schemes; they tend to provide rather low accuracy, even for nearly uniform airfoil boundary discretizaton. An alternative approach is to transform the original problem into a BIE with bounded or at least absolutely integrable kernel. Such approach combined with the Galerkin or Petrov — Galerkin method makes it possible to develop a hierarchy of numerical schemes (called "*T*-schemes") for accurate BIE solution even for coarse and essentially non-uniform discretization. It is shown that these schemes provide the first and second orders of accuracy (for piecewise-constant and piecewise-linear solution representation, respectively) on smooth airfoils.

In case of airfoils with sharp edges or corner points, the BIE solution has weak singularity that cannot be reconstructed correctly in the framework of the mentioned Galerkin-type schemes; their order of accuracy becomes not higher than the first one. To overcome this issue, a numerical scheme is suggested that allows for solution singularity resolving and provides the 2-nd order of accuracy. As a model problem, the added mass tensor components computation is considered, since its exact value is known for the Joukowsky wing airfoil with sharp edge (cusp point).

It should be noted that the developed schemes make it possible to simulate laminar flow regimes around airfoils with high accuracy. For higher Reynolds numbers, correct results for flow simulation are observed only for airfoils with sharp edges and corner points and additionally in the case of the most intensive flow separations nearby these points. The reason for inaccurate results for other flow regimes is connected with incorrect modeling of separation from smooth airfoil surface at high Reynolds numbers. This study proposes some ideas how to solve this problem and to develop new modification of vortex methods that would be applicable to modeling turbulent effects.

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A higher–order time domain boundary element formulation based on isogeometric analysis and the convolution quadrature method

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Keywords: Convolution quadrature method, Higher–order boundary element method, Isogeometric analysis

A higher–order isogeometric boundary element method (IGA–BEM) is presented to solve scattering problems governed by the scalar wave equation as in acoustics and the elastodynamic equation for linear elastic waves in an isotropic homogeneous medium. The underlying boundary integral equations imply time–dependent convolution integrals and allow us to determine the sought quantities in the bounded interior or the unbounded exterior of the scatterer's surface after solving for the unknown Cauchy data [1]. Reducing the problem to the scatterer's surface motivates to accurately discretize the possibly curved spatial manifold by a higher–order isogeometric parametrization. In the present work, the time–dependent convolution integrals are approximated by multi–stage Runge–Kutta based convolution quadratures [2] that are based on steady–state solutions in the Laplace domain. The Runge–Kutta convolution quadrature method enables high convergence rates in time matching the convergence rates of the higher–order isogeometric approximation of the solutions in the spatial variable. With the extension to higher–order isogeometric analysis a smooth basis of the solution space is provided using NURBS basis functions and a high continuity of the solution is ensured within patches [3]. This improves the approximation of the wave radiation without relying on fine meshes. Furthermore, using an isogeometric boundary representation and NURBS basis functions as the basis for approximating the solution opens the door to perform simulations on models from Computer Aided Design (CAD). We study a symmetric Galerkin variational formulation for mixed problems [4] worked out on various manifolds. We investigate space–time convergence rates and examine the accuracy of the used integration routine. Key points regarding the implementation of the higher–order isogeometric analysis into existing classical lowest–order BEM codes are outlined and explained.

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Partial Integration based Regularization in Fast Multipole Boundary Element Method

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Keywords: singular integrals, Stokes' theorem

The boundary element method (BEM) has proven to be a highly effective numerical method for solving specific elliptic boundary value problems, particularly advantageous for complicated geometries and (semi-) infinite computational domains. Despite its benefits, BEM faces significant challenges in dealing with computationally expensive dense system matrices and the presence of singular integral kernels. The issue of BEM being computationally expensive can be handled with certain fast methods. One way to reduce memory and computational cost is by using the Chebyshev interpolation-based fast multipole method (FMM) [1]. This subdivides the computational domain into a far-field, where Chebyshev interpolation approximates the kernel, and a near-field, where singularities must be ad- dressed. To handle singular kernels, there exists several analytical/numerical integration or regularization techniques. One such regularization technique combines partial integration with Stokes' theorem to transform hyper-singular and strong singular integral kernels into weakly singular ones [2, 3]. How- ever, due to the subdivision of the domain by FMM, the line integral terms from Stokes' theorem must be computed as the geometry can no longer be considered a closed surface.

This work presents two techniques to address this issue. The first technique computes the additional line integrals near the boundary of the near- and far-field, demonstrated for a 3D-Helmholtz problem using Galerkin BEM. The second technique applies regularization to all integral kernels before employing FMM operators, necessitating the computation of additional FMM operators, illustrated for a 3D linear elastostatics problem using collocation BEM. Numerical results are provided to validate the effectiveness of the proposed methods.

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Application of boundary integral quadrature method to torsion problems of the orthotropic bars and its treatment of degenerate scale problem

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Keywords: boundary integral quadrature method, orthotropic bar, adaptive exact solution, Gaussian quadrature, CLEEF

Regarding the Saint-Venant torsion problem of orthotropic bars, the boundary integral quadrature method (BIQM) is employed to solve the stress function $\Phi(x)$ and warping function $W_a(x)$. We introduce the adaptive exact solution into the boundary integral eqation. Not only the singular integral in the sense of the Cauchy principal value can be novelty determined but also the calculation of solid angle on the boundary is free. After using the parametric form to represent the boundary contour and adopting the Gaussian quadrature for the boundary integral equation, the boundary integral equation is nothing more than an algebraic equation. Therefore, only collocating Gaussian points on the boundary are required to obtain the simultaneous equation. Finally, we calculate the torsional rigidity and the boundary shear stresses with different cross-sections by using the BIQM. After comparing the present results to these of references [3-5], agreement is made. In addition, the numerical instability due to the degenerate scale for the orthotropic case is dealt with by using the combined Laplace integral equation formulation (CLEEF) method.

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A boundary element method for the magnetic field integral equation in electromagnetics

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Keywords: magnetic field integral equation, electromagnetic scattering, boundary element method

The magnetic field integral equation (MFIE) is widely used in practical applications to model time-harmonic electromagnetic scattering by perfectly conducting bodies. Despite its utility, attention paid to the MFIE is not at the same level with those to other boundary integral equations, such as the electric field integral equation and combined field integral equations. Moreover, most of the papers devoted to boundary element methods for the MFIE have been contributed by practitioners. A rigorous numerical analysis is therefore needed!

On smooth surfaces, the governing MFIE operator is well-known to be a Fredholm operator of second kind. Provided that the wave number is not an eigenvalue of the interior Maxwell's problem, the continuous variational problem of the MFIE on smooth domains has been shown to be uniquely solvable [1]. In the discrete setting, however, the unique solvability does not necessarily hold true. More particularly, the Galerkin discretization of the identity operator (the principal part) occurring in the MFIE using lowest-order Raviart-Thomas elements has been proven to not satisfy the discrete inf-sup condition [2]. As a result, the coercivity of the corresponding Galerkin discretization of the MFIE does not hold true.

In this contribution, we intend to provide rigorous analysis on a boundary element method for the MFIE on polyhedra. On non-smooth Lipschitz surfaces, the average of exterior and interior double layer boundary integral operators is generally not compact, rendering the existing result for smooth surfaces not applicable. We firstly prove that the governing MFIE operator is compact perturbation of a T -coercive operator (different from the identity operator). By means of a Fredholm alternative argument, this property implies the well-posedness of the continuous variational problem of the MFIE on Lipschitz domains.

Then, we analyze the mixed discretization scheme for the MFIE which was introduced in [3]. This scheme employs Raviart-Thomas basis functions for the solution space and Buffa-Christiansen functions for the test space to exploit the duality between the two discrete spaces. In typical appli- cations, the mixed discretization fulfills the discrete inf-sup condition [3]. However, on some singular geometries, the lower bound of the discrete duality constant gives rise to the lack of the discrete inf-sup condition. We provide a condition solely dependent of the geometry under which the discrete inf-sup condition can be proven. Once the discrete inf-sup condition is fulfilled, the asymptotic quasi- optimality of numerical solutions is obtained. In addition, the resulting matrix system can be shown to be well-conditioned regardless of the boundary mesh refinement.

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Decomposed B-FNO for 3D parametric acoustic wave analysis

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Keyword: Surrogate model; Acoustic wave equation; Boundary element method; Fourier neural operators.

Wave analysis is crucial in various applications such as noise control and medical treatment, often necessitating parametric analyses across different settings. Traditional numerical methods become computationally impractical for these tasks due to the vast number of simulations involved, especially in unbounded domains. The data-driven surrogate modelling approach, B-FNO, which integrates BEM and Fourier neural operator (FNO) is proposed to handle these problems. This approach establishes a mapping between the boundary geometry and physical space to the BEM boundary solution space. The B-FNO retains the advantage of BEM in effectively addressing infinite domain problems while simultaneously enhancing the computational efficiency of BEM during variations in geometric frequency and other parameters. Although B-FNO exhibit a time complexity of $O(N\log N)$ same as fast BEM, and maintain a stable advantage in terms of overhead, they still encounter limitations in efficiency and memory usage when applied to large-scale three-dimensional problems. In this study, we introduce a decomposed B-FNO. Drawing inspiration from matrix decomposition techniques, we replace the original input undergoing 2D FFT with a series of 1D vector multiplications. The critical insight here is that the precise fidelity of the sum of the vector products to the original matrix is not important. Instead, the focus is on ensuring that the overall process accurately captures the essential patterns required for reliable predictions. When the original input size for the 3D B-FNO is $M \times N$, the time complexity is significantly reduced from $O(MN\log MN)$ to $O(MN)$. We present various examples of radiation and scattering to demonstrate the guaranteed accuracy and improved efficiency of this decomposed B-FNO. Additionally, we consider complex boundary conditions and large-scale geometries to illustrate the capability of the decomposed B-FNO in addressing engineering challenges effectively.

Robust topology optimization in fully coupled vibro-acoustic systems with random and interval variables based on FEM-BEM

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Keywords: robust topology optimization, vibro-acoustic system, hybrid uncertainty, polynomial chaos-Chebyshev interval method, multiresolution topology optimization

Research on topology optimization for fully coupled vibro-acoustic systems has been recently carried out [1, 2] based on the coupled finite element method-boundary element method (FEM-BEM). In those optimization processes, the parameters of geometry, load, environment, and material properties are assumed to be deterministic. However, in engineering practice, these parameters are usually uncertain. Therefore, it is essential to consider these uncertainties, which can be modeled as both random and interval models, in the analysis to obtain a more realistic numerical solution for the analyzed structures [3].

In this study, a robust topology optimization method is proposed for fully coupled systems comprising external acoustic fields and elastic structures. The polynomial chaos-Chebyshev interval (PCCI) method in combination with the coupled FEM-BEM is employed to calculate the hybrid uncertain response. A model reduction method based on proper orthogonal decomposition is utilized in con- junction with PCCI to reduce the computational cost in uncertainty quantification.

A weighted sum of the upper bounds of the mean and standard deviation of the radiation sound power level is set as the objective function for robust topology optimization. Based on the multiresolution topology optimization (MTOP) approach proposed by Nguyen et al. [4], the optimization model is decoupled into the analysis mesh, design mesh and density mesh. A coarser mesh is utilized in FEM analysis, which can save the computational cost. Meanwhile, finer meshes are applied for the density mesh and the design variable mesh, enabling the generation of high-resolution topological designs.

Numerical examples demonstrate that the proposed robust topology optimization method can obtain designs that are less sensitive to hybrid uncertainties compared to deterministic topology optimization, which neglects the uncertain parameters. Via the MTOP, a relatively low computational cost is needed to obtain high-resolution robust designs.

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T-scheme for solution of the boundary integral equations in vortex methods for three-dimensional flows simulation

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Keywords: vortex methods, three-dimensional flow, boundary integral equation, double layer

Lagrangian vortex methods are considered for 3D flows simulation around bodies and system of bodies, which can be movable. Vortex methods make it possible to calculate hydrodynamic loads acting on the structures in the flow with acceptable accuracy at rather low computational cost, that allows for solving coupled FSI problems and predicting structures behavior in unsteady regimes.

The most important and time-consuming operations in vortex methods are connected with simulation of vorticity motion in the flow and its generation on body surface. The first problem is similar to the classical *N*-body problem and can by efficiently solved by using approximate fast algorithms. For the second operation an original approach is developed, called "*T*-scheme". It is based on numerical solution of the vectorial weakly-singular boundary integral equation with respect to vortex sheet intensity. Due to use of the Galerkin method, suggested approach is robust and rather accurate even for piecewise-constant solution representation; it successfully works with surface meshes of low quality.

The kernel of the BIE is the gradient of fundamental solution of the Laplace equation. Semi-analytic approach is suggested for calculation of the repeated integrals over mesh cells (its early version was presented [1]). For integration over cells with common edge or vertex singularity additive exclusion is done that allows for integrals computation with high accuracy. The presented approach is not as universal as [2] and [3], but is much faster in practice. To provide the preliminarily specified accuracy of numerical integration, the Runge rule is used. The algorithm for integrals computation is implemented as C++ library (both for CPU and GPU), that shows good performance.

In addition, the least-squares method is implemented for the double layer potential density reconstruction, that is required for further flow simulation and consistent vortex sheet transformation into the system of closed vortex loops [4]. Since non-solenoidal "attached" vorticity, that is required for simulation of rotational motion of the body, cannot be represented with vortex loops, special approach is also developed for its influence accounting.

High quality of the BIE solution makes it possible to provide high quality of velocity field reconstruction nearby the body surface that in turn plays a key role in correct calculation of hydrodynamic loads. The suggested algorithms are implemented in tho original code VM3D; number of model problems are considered. Acceptable agreement with experimental data is observed. Directions of further improvement of the scheme and algorithms are discussed.

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Identification of non-Hermitian degeneracy in acoustic scattering systems using boundary integral equations

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Keywords: Non-Hermitian degeneracy, Exceptional point, Helmholtz equation, Scattering resonance

It is well known that some wave systems exhibit a special type of eigenvalue degeneracy called *non-Hermitian degeneracy*. Roughly speaking, non-Hermitian degeneracy is the simultaneous coalescence of two (or more) eigenvalues and their corresponding eigenstates. For instance, let $\{A_\rho\}$ be a family of finite-dimensional square matrices labeled by a complex parameter ρ, and let $\lambda_\rho \in \mathbb{C}$ be an eigenvalue of A_ρ with algebraic multiplicity $m \geq 1$. We say that λ_ρ is non-Hermitian degenerate if its geometric multiplicity is not equal to m, i.e., $\dim N\,(\lambda_\rho I - A_\rho) \neq m$, where N denotes the null space. Such parameters $\rho \in \mathbb{C}$ are called *exceptional points* [1].

In this study, we present numerical evidence that such non-Hermitian degeneracy occurs in two-dimensional scattering systems. Specifically, we consider Helmholtz' equation in the exterior of a union of some bounded domains with homogeneous Neumann boundary condition (Figure 1). Under the convention $e^{-i\omega t}$ with angular frequency ω, a scattering resonance is a pole of the Fredholm operator-valued analytic function $\omega \to \frac{1}{2}I - K_\omega$ in lower half-plane, where K_ω denotes the double-layer boundary integral operator [2]. We adopt a Nyström method [3] and investigate the multiplicity of a scattering resonance numerically. In addition, some anomalous phenomena are observed, e.g., square-root sensitivity of scattering resonances.

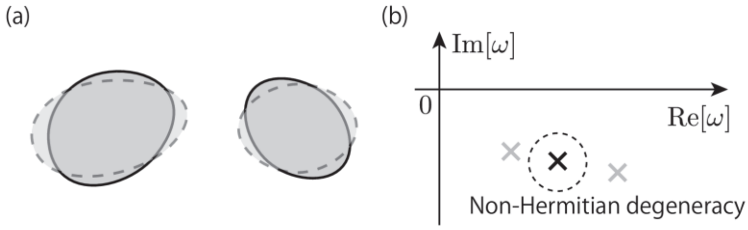

Figure 1: (a) Perturbation of the shape of scatterers in two dimensions. (b) Distribution of scattering resonances corresponding to the two shapes.

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Fast Direct Solvers for Boundary Integral Equations

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Keywords: Boundary Integral Equation; Fast Direct Solver; Rank-structured matrix; H-matrix; Randomized Algorithms.

That the linear systems arising upon the discretization of boundary integral equations associated with elliptic PDEs can be solved efficiently is well established. In particular, combining iterative solvers with fast algorithms for the matrix-vector multiplication such as the Fast Multipole Method has have proven to be very successful, and often attain linear overall complexity for the solve. Interestingly, it has recently been demonstrated that it is often possible to directly compute an approximate inverse to the coefficient matrix in linear (or close to linear) time [1–3, 5]. The talk will argue that such direct solvers have several compelling qualities, including improved stability and robustness, the ability to solve certain problems that have remained intractable to iterative methods, and dramatic improvements in speed in certain environments.

After a brief introduction to the accelerated direct solvers for BIEs, particular attention will be paid to a set of recently developed randomized algorithms that construct data sparse representations of discretized integral operators [4, 6]. These algorithms are entirely black box, and interact with the linear operator to be compressed only via the matrix-vector multiplication. This opens up the possibility to deploy fast algorithms for matrix inversion in any environment where a user already has access to a method (such as the Fast Multipole Method) for matrix-vector multiplication. These techniques have also proven powerful for resolving FEM-BEM coupling problems, and other situations where pre-conditioners are difficult to obtain.

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Fundamental Study on Numerical Homogenization for Wave Problems Using Self-consistent and Boundary Element Methods

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Keywords: self-consistent method, boundary element method, homogenization, elastic wave

For ultrasonic nondestructive testing of infrastructures, it is necessary to consider wave propagation in concrete media. Since concrete is an inhomogeneous material, the behavior of elastic wave propagation in it is more complex compared to homogeneous materials. In particular, scattering attenuation and the disturbance of wave velocity due to coarse aggregates significantly influence ultrasonic measurements. At this stage, the concrete material is treated as a homogeneous material with measured wave velocity and attenuation, which are then used for flaw imaging. Therefore, it is important to estimate the effects of inhomogeneity on wave propagation characteristics for designing experimental measurements, such as frequency selection. Given the known mix design of concrete and the particle size distribution of aggregates, it is likely possible to estimate the macroscopic material properties of concrete.

Estimation of macroscopic material constants for inhomogeneous materials requires some homogenization method. An Effective Field Method (EFM) [1] is one of the effective techniques for wave problems. In ultrasonic testing, it is necessary to estimate the wave velocity and scattering attenuation of elastic waves. Furthermore, it is desirable to estimate not only the average physical quantities but also the variance due to the randomness of the aggregate particle size distribution and spatial distribution.

This study aims to estimate macroscopic material constants for wave propagation using the EFM. Since the original EFM [1] is a formulation that excludes variance and obtains only the average material constants, the formulation for evaluating material constants is modified. The boundary element method is used to solve wave scattering by inclusions, which is required in the process of EFM. Although it is necessary to consider a three-dimensional elastic wave field, a two-dimensional problem is treated as a fundamental study.

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A fast direct solver for time-harmonic in-plane elastic wave scattering problems

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Keywords: Elastodynamics, Boundary integral equation, Fast direct solver

1 Introduction

In elastodynamics, partial differential equations that describe waves propagating through solids are well-known, and they have important applications in fields such as ultrasonic non-destructive testing. These equations are often solved under the assumption of bounded inclusions or cavities within an infinite domain, making the boundary element method (BEM) a promising approach.

Because the BEM requires at least $O(N^2)$ complexity, several fast solvers are developed. The best known method is the FMM combined with an iterative solver such as GMRES [1]. However, when there are many incident waves in a problem setting that are discretized into the same coefficient matrix, this method requires k times the computation time for k incident waves.

Even in cases with multiple right-hand sides, the LU decomposition of the H-matrix, a low-rank approximation of the off-diagonal part of the coefficient matrix, is known as a fast direct solver (FDS) and has been applied to three-dimensional elastodynamic problems [2]. However, the computational complexity of existing research is at best $O(N(\log N)^2)$, and because they used the collocation method, it is not easy to properly handle hypersingular integral operators. In the field of boundary element methods, methods using hypersingular integral operators have been developed to avoid fictitious eigen-value problems.

2 Method

We apply the Martinsson-Rokhlin type FDS [3] to in-plane elastic wave scattering problems under a Galerkin method, allowing C0-class basis functions, enables the construction of a fast direct solver with $O(N)$ complexity, high parallelization efficiency, and simple implementation. The Martinsson-Rokhlin method is known to achieve $O(N)$ complexity, at least for two-dimensional Helmholtz boundary value problems [4].

The presentation will explain the formulation and details of the method, and will demonstrate the validity and advantages of the method through numerical examples.

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Collocation and a Mixed Approximation of the Boundary Element Method for Linear Elasticity

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Keywords: Boundary element method, Collocation, Mixed Approximation, Hierarchical matrices, Linear elasticity

Using the boundary element method to solve mixed boundary value problems generally results in non-sparse matrices. To reduce storage, we approximate the involved operators by hierarchical matrices [1, 2].

Furthermore, to speed up simulations we would like to replace the Galerkin method by collocation for the discretisation of the operators.

The standard formulation of the boundary integral equations involves a hypersingular integral operator, which is not defined in the case of collocation. In order to avoid this operator, we will use a mixed approximation which was introduced by Olaf Steinbach for the Laplace equation [3]. With the help of the Steklov-Poincar operator a coupled saddle point problem can be derived only involving single and double layer potential operators.

Our aim is the application of this mixed formulation together with collocation to the Lamé equation from linear elasticity in order to accelerate the simulation.

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Calculation of singular integrals in Galerkin boundary element methods

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Keywords: Singular integrals, Galerkin BEM, 3D

In this talk, we present closed formulas for the computation of the matrix entries in Galerkin boundary element methods (BEM) in 3D. The entries require the integration of singular kernels functions over pairs of surface panels, which becomes difficult when the panels have non-empty intersection. Although the singularities can be removed by means of coordinate transformations, the approximation by cubature rules is still expensive due to the high-dimensionality of the integrals. We propose an alternative approach based on the analytical calculation of the integrals for the standard Galerkin discretization of the Laplace equation with piece-wise constant and linear boundary elements on flat triangles. We show that the regularized integrals obtained by the Duffy transformation admit closed and exact formulae for identical triangles and triangles with a common edge. For the remaining cases, we reduce the four-dimensional integrals to one- or two-dimensional integrals. In this way, we are able to compute the matrix entries accurately while reducing the computational costs compared to numerical integration. We verify the correctness of the formulae and demonstrate their efficiency in numerical experiments.

Analysis for Complex Plane Cracks Using the Singular Integral Equation Method

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Keywords: complex plane crack, stress intensity factor, singular integral equation, boundary element method

A singular integral equation method is proposed to analyze the complex plane cracks in one-dimensional (1D) orthorhombic quasicrystal. Using the Somigliana formula, a set of singular integral equations of the curved crack is derived. Based on the general situation of the curved crack, the singular integral equations of the inclined and circular crack are given. Then the analytical solutions of the singular phonon and phason stresses and the stress intensity factors (SIFs) near the tip of the inclined crack and circular crack are obtained. Gauss-Chebyshev integral formula is introduced to calculate the singular integral equation, and a numerical algorithm for solving the stress intensity factor is proposed. Numerical solutions for the phonon and phason stress intensity factors of some examples are solved and discussed.

FMM and H-matrix based approaches within a 3D-ACA accelerated time domain boundary element method

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Keywords: FMM, 3D-ACA, time domain

The time domain Boundary Element Method (BEM) for the homogeneous wave equation with vanishing initial conditions is considered. The generalized convolution quadrature method (gCQ) developed by Lopez-Fernandez and Sauter [3] is used for the temporal discretisation. The spatial discretisation is done classically using low order shape functions. A collocation approach is applied for the Dirichlet problem and a Galerkin approach for the Neumann problem.

Essentially, the gCQ requires to establish boundary element matrices of the corresponding elliptic problem in Laplace domain at several complex frequencies. Consequently, an array of system matrices is obtained. This array of system matrices can be interpreted as a three-dimensional array of data which should be approximated by a data-sparse representation. The multivariate Adaptive Cross Approximation (3D-ACA) [1] can be applied to get a data sparse representation of these three- dimensional data arrays. Adaptively, the rank of the three-dimensional data array is increased until a prescribed accuracy is obtained. On a pure algebraic level it is decided whether a low-rank approximation of the three-dimensional data array is close enough to the original matrix. Within the data slices corresponding to the BEM calculations at each frequency either the standard H-matrices approach with ACA [2] or a fast multipole (FMM) approach can be used. The third dimension of the data array represents the complex frequencies. Hence, the algorithm makes not only a data sparse approximation in the two spatial dimensions but detects adaptively how much frequencies are necessary for which matrix block.

In the presentation, this methodology is recalled and both versions either using H-matrices in the slices or FMM will be compared. The study is numerically performed at selected examples as the mathematical analysis gives the same complexity. Nevertheless, the performance of the algorithm differs.

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Performance evaluation of the Calderon preconditioning for the Burton–Miller boundary element method in three-dimensional transmission problems

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Keywords: transmission problem, boundary integral equation, Burton–Miller method, Calderon's preconditioning

In this study, we introduce a 3D implementation of a recently proposed Calderon-preconditioned boundary element method (BEM) based on the Burton–Miller (BM) formulation for transmission problems [1].

It has been a long-standing issue that a naive use of the Burton–Miller method known as an accurate BEM for exterior problems to the transmission problems gives ill-conditioned boundary integral equations. On the other hand, it is well known that the Calderon preconditioning is effective in improving the condition of algebraic equations in transmission BEMs such as the one with the PMCHWT formulation [2]. In [1], Matsumoto et al extended the Calderon preconditioning to the collocation BEM based on the BM formulation for the transmission problems, in which its simple implementation is presented; one just needs to reorder the integral operator multiplied by a constant and slightly modify some of them, resulting in a reduction in the Krylov iterations applied to the algebraic equations derived from the underlying integral equation.

This paper extends the method [1] to three-dimensional problems and evaluates its performance. In the numerical experiments, we consider the transmission problem where a scalar plane wave in a host medium with wave velocity $c = 1$ travelling along x_2 axis is scattered by a unit sphere filled with a material of $c = 1/\sqrt{2}$. The left of Figure 1 shows the eigenvalue distribution of the preconditioned BEM matrices of naive and proposed formulations for the case of the angular frequency $\omega = 1$. One finds that the eigenvalues of the proposed method accumulate at a single point in the complex plane. The centre of Figure 1 shows the number of GMRES iterations for the scattering analyses versus the BEM degrees of freedom N for the case of $\omega = 3$. The proposed method requires less iterations than the conventional one. It is also noteworthy that the performance of the proposed method is less sensitive to N. We also confirmed, from the right of Figure 1 showing GMRES iterations vs N for the scattering by concentric spheres whose radius are 1 and 2 each of which is respectively composed of a material with $c = 1/\sqrt{2}$ and $1/\sqrt{3}$, that the proposed method is also efficient for multi-material problems

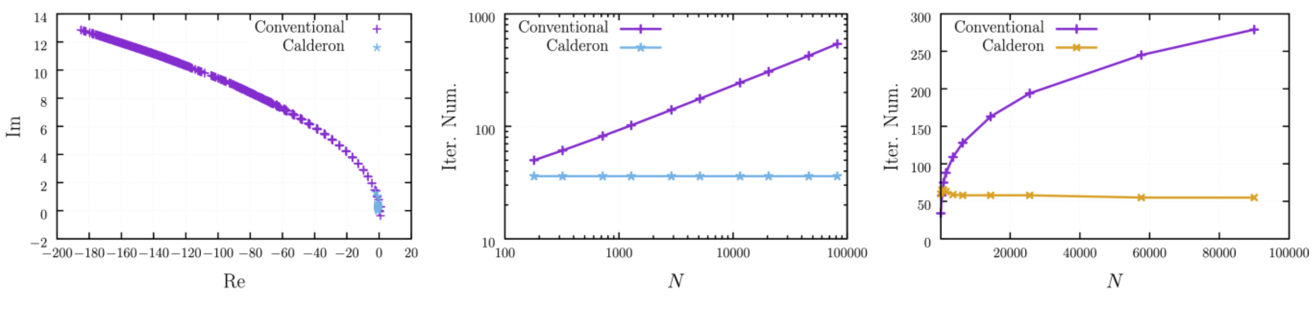

Figure 1: (left) The eigenvalue distribution of BEM matrices, (centre) dependence of GMRES iterations on the degrees of freedom N, and (right) GMRES iterations vs N in multi-materials.

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Structural optimisation for acoustic cavity problems in half space using a hybrid boundary integral representation

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Keywords: BEM, Structural optimisation, Cavity scattering in half space, CMA-ES

A recently proposed numerical method with a hybrid boundary integral representation that com- bines the Sommerfeld integral and layer potentials [1] is now considered promising in solving scattering problems in half space. Our previous study [2] demonstrated that the method could also address cavity scattering involving infinite-length boundaries with local perturbations of a concave shape by introducing some virtual boundaries enclosing the cavity. In the present contribution, we further explore the application of the method to a structural optimisation problem via CMA-ES aiming to manipulate the cavity scattering.

We here consider a cavity scattering defined in $\mathbb{R}^2_+ \cup \Omega_{\text{cavity}} \setminus \overline{\cup_i \Omega_i}$ with $\mathbb{R}^2_+ := \{\boldsymbol{x} \mid y > 0\}$, $\Omega_{\text{cavity}} := \{\boldsymbol{x} \mid y < 0,\ x^2 + y^2 < 1\}$, and $\Omega_i := \{\boldsymbol{x} \mid (x - c_i)^2 + (y - d_i)^2 < r^2\}$, where (c_i, d_i) and r_i respectively denote the centre and radius of i^{th} cylindrical scatterer. Our objective is to maximise the sound intensity P in a squared area $\{\boldsymbol{x} \mid -0.25 \leq x \leq 0.25,\ 3.25 \leq y \leq 3.75\}$ in front of the cavity by optimising the location c_i, d_i, and the size r_i of the scatterers. To this end, we minimise the objective function $J = 100 - P$ via CMA-ES.

We here show an illustrative numerical example. In this example, we fixed the centres of three cylinders as $(c_i, d_i) = (-0.5, 0.0)$, $(0.5, 0.0)$, and $(0.0, 1.0)$ and set their radii r_1, r_2, and r_3 as the design variables. The point source is arranged at $(0.0, -0.5)$ in the cavity. Figure 1 (left) shows the history of the objective function J. One observes that J is converged and minimised significantly. The centre and right figures of Figure 1 show the sound intensity around the cavity with unoptimised and optimised cylinder allocations, respectively. These figures show that P was appropriately maximised by the proposed optimisation strategy.

In the oral presentation, we shall present some other numerical results along with the detailed formulation of the proposed method focusing on computational efficiency.

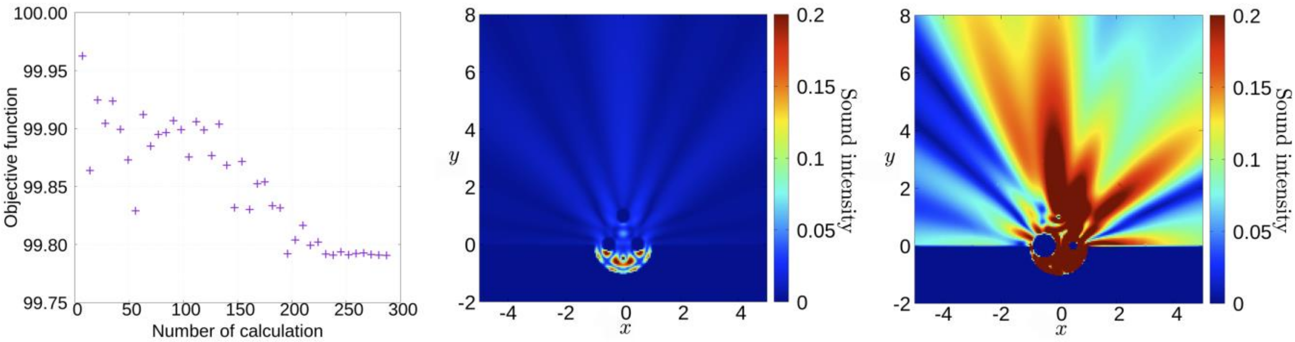

Figure 1: History of the objective function (left). Sound intensity around the cavity for the unoptimised (centre) and optimised (right) cylinder allocations.

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Fast acoustic singular boundary method based on directional H^2-matrices

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Keywords: singular boundary method, directional H2-matrices, semi-analytical, acoustic, High-frequency

The paper presents the application of directional H2-matrices [1] in conjunction with the semi-analytical meshless singular boundary method [2] for solving high- frequency acoustic problems. In the singular boundary method, fundamental solutions are used as trial functions, inherently satisfying the boundary conditions at infinity when addressing acoustic issues. By introducing origin intensity factors, the singularity of the fundamental solution at the source node is effectively eliminated, avoiding the need to evaluate singular and near-singular integrals. However, when applied to high-frequency problems, the SBM based on standard hierarchical matrices encounters challenges in achieving low-rank approximations due to the high oscillatory of the fundamental solutions. Directional compression techniques are used to solve this problem by using decompositions based on plane waves. The numerical results show that the combination of directional H2 matrices and the singular boundary method can successfully solve the high-frequency Helmholtz equation.

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A multi-scaled fracture analysis method based on BEM and PD

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Keywords: multi-scale, BEM, PD, fracture analysis

A multi-scale method based on a combination of the boundary element method (BEM) and peridynamics (PD) has been verified as an efficient approach to modeling the crack propagation problems. The special feature of this method is that it can take full advantage of both the BEM and PD to achieve a higher level of accuracy and efficiency. In the implementation process, based on the scale of the structure and the location of cracks, the considered domain can be divided into non-cracked parts and cracked parts. However, for actual application, the crack propagation path cannot be predetermined. Thus, an adaptive coupling technique is necessary. In this paper, an adaptive coupling approach based on BEM-PD has been developed. A small PD domain is predesigned on the crack tips or automatically created at the largest stress part. The remaining part is modeled by the BEM. The iterative algorithm is employed by transforming the force equilibrium condition at the interfaces between two models to achieve the displacement continuously. The PD domain always moves with the crack tip moving, the created macro cracks in the PD domain will be replaced by the crack edges of BEM. This method can be used to predict the crack propagation, additionally, controlling the PD domain in a very small region is an important way to improve the efficiency of BEM- PD.

Synthetic seismograms in liquid-solid coupled layered media

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Keywords: dislocation, ocean water, layering, elastic anisotropy, system of vector functions, Love number, seismograms, Green's function

Earthquakes could occur under the ocean as well as inland. Such an example is the recent earthquake happened offshore in Hualian, Taiwan. While many research works have been carried out on synthetic seismograms in layered solids, those from the liquid-solid coupled layered media are still missing. In this paper, a novel and comprehensive method is proposed for calculating the dislocation Love numbers (DLNs), Green's functions (GFs), and the corresponding deformation in a transversely isotropic and layered elastic half-space which is overlain by layered ocean water. The solution is based on the newly introduced Fourier-Bessel series system of vector functions and the dual-variable and position method, coupled with the dislocation source functions. Numerical results show not only the effect of ocean what on the wave response on the seabed, but also the complicated interaction between ocean and the soil below the ocean. DLNs, GFs and the corresponding dynamic deformation fields are presented to illustrate the effect of the ocean water on these dynamic responses.

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